NEW RESULTS ON QUANTUM CHAOS IN ATOMIC NUCLEI

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In atomic nuclei, ordered and chaotic states generally coexist. In this paper the transition from ordered to chaotic states will be discussed in the framework of roto–vibrational and shell models. In particular for ^{160}Gd, in the roto–vibrational model, the Poincarè sections clearly show the transition from order to chaos for different values of rotational frequency. Furthermore, the spectral statistics of low–lying states of several fp shell nuclei are studied with realistic shell–model calculations.

1 Introduction

In atomic nuclei, just as in other many–body systems, ordered and chaotic states [a] generally coexist. In fact, in zero–order approximation the relevant elementary excitations (such as rotation, vibration and single–particle) may be regarded as independent modes. Then we consider the interaction between these elementary modes[1)]. At one end of the chain of complexity we have a single mode which can be considered "regular", whereas at the other end there are the so–called "stochastic" or "chaotic states".

In order to discuss the coexistence in atomic nuclei of ordered and chaotic states, many models have been used[2),3)] but, for reasons of space, we shall mention only the roto–vibrational and shell models.

[a] Here we use the term chaos for a quantal system in a very restricted sense: "*Quantum Chaology is the study of semiclassical behaviour characteristics of systems whose classical motion exhibits chaos*". If the semiclassical limit is not known we use spectral statistics[4)] (e.g. $P(s)$ and $\Delta_3(L)$) to distinguish between ordered and chaotic states.

2 The Roto–Vibrational Model

This model has been amply described in Ref. 5, 6 and 7 and we limit ourselves to reporting only a few basic formulae. The hamiltonian is

$$H = H_{vib} + H_{rot}, \tag{1}$$

where

$$H_{vib} = \frac{1}{2}B(\dot{a}_0^2 + 2\dot{a}_2^2) + V(a_0, a_2), \tag{2}$$

$$H_{rot} = \frac{1}{2}\sum_{k=1}^{3}\omega_k^2 J_k(a_0, a_2), \tag{3}$$

with

$$V(a_0, a_2) = \frac{1}{2}C_2(a_0^2 + 2a_2^2) + \sqrt{\frac{2}{35}}C_3 a_0(6a_2^2 - a_0^2) + \frac{1}{5}C_4(a_0^2 + 2a_2^2)^2 + V_0. \tag{4}$$

The shape of the nuclear potential $V(a_0, a_2)$ is function of C_2 and $\chi = C_3^2/(C_2C_4)$[7]. The parameters a_0 and a_2 are connected to the deformation β and asymmetry γ by the relations

$$a_0 = \beta\cos\gamma, \quad a_2 = \frac{\beta}{\sqrt{2}}\sin\gamma. \tag{5}$$

In terms of the new variables the components of the moment of inertia are[8]

$$J_k = 4B\beta^2\sin^2(\gamma - \frac{2\pi}{3}k). \tag{6}$$

If the nucleus has an axially symmetric equilibrium configuration and

$$\omega_1 = \omega_2 = \frac{\omega}{\sqrt{2}}, \quad \omega_3 = 0, \tag{7}$$

the hamiltonian (1) can be written as

$$H = \frac{1}{2}B(\dot{a}_0^2 + 2\dot{a}_2^2) + V(a_0, a_2) + \frac{1}{2}B\omega^2(3a_0^2 + 2a_2^2), \tag{8}$$

where $V(a_0, a_2)$ is given by (4) and V_0 is chosen to have the minimum of the potential equal to zero.

Figure 1: The Poincarè sections for ^{160}Gd at the energy 5.5 MeV and for different values of rotational frequency; from the top: $\hbar\omega = 0$ MeV, $\hbar\omega = 0.5$ MeV, $\hbar\omega = 1$ MeV. Adapted from Ref. 9.

3 Numerical study of the order–chaos transition

A very useful tool for the study of the global instability is provided by the Poincarè sections. The classical trajectories have been calculated by a fourth order Runge–Kutta method[9].

The Hamilton equations are as follows

$$\dot{a}_0 = Bp_0,$$

$$\dot{a}_2 = 2Bp_2,$$

$$\dot{p}_0 = -C_2 a_0 - 2\sqrt{\frac{2}{35}} C_3 (3a_2^2 - 3a_0^2) - \frac{4}{5} C_4 a_0 (a_0^2 + 2a_2^2) - 3B\omega^2 a_0, \quad (9)$$

$$\dot{p}_2 = -2C_2 a_2 - 12\sqrt{\frac{2}{35}} C_3 a_0 a_2 - \frac{8}{5} C_4 a_2 (a_0^2 + 2a_2^2) - 2B\omega^2 a_2,$$

where p_0 and p_2 are the conjugate momenta

$$p_0 = B\dot{a_0}, \quad p_2 = 2B\dot{a_2}. \quad (10)$$

Figure 1 shows the Poincarè sections for ^{160}Gd at the energy 5.5 MeV and for different values of rotational frequency. The figure clearly shows a chaos–order transition as the frequency ω increases. In Figure 2, for ^{166}Er, the Poincarè sections are shown for different values of the energy and rotational frequency $\omega = 0$. As can be seen, there is a chaos–order transition, but not so sharp as in the previous case.

Fluctuation properties of quantal systems with underlying classical chaotic behaviour and time–reversal symmetry are in agreement with the predictions of the Gaussian Orthogonal Ensemble (GOE), and quantum analogs of classically integrable systems display the characteristics of Poisson statistics[10]. In general, various statistics may be used to show the local correlations of the energy levels; we shall discuss $P(s)$ and $\Delta_3(L)$ only. $P(s)$ measures the probability that two neighbouring eigenvalues are a distance "s" apart. For GOE we have the Wigner distribution

$$P(s) = \frac{\pi}{2} s \exp\left[-\frac{\pi}{4} s^2\right], \quad (11)$$

which gives level repulsion. $\Delta_3(L)$ is defined for a fixed interval $(-L/2, L/2)$ as the least–square deviation of the staircase function $N(E)$ from the best straight line fitting it

$$\Delta_3(L) = \frac{1}{L} \min_{A,B} \int_{-L/2}^{L/2} [N(E) - AE - B]^2 dE, \quad (12)$$

Figure 2: The Poincarè sections for ^{166}Er at the rotational frequency $\omega = 0$ and for different values of the energy: (a) $E = 1$ MeV, (b) $E = 6$ MeV, (c) $E = 9$ MeV, (d) $E = 12$ MeV. Adapted from Ref. 9.

Figure 3: Spectral statistics $P(s)$ and $\Delta_3(L)$ for ^{166}Er at the rotational frequency $\omega = 0$ for different energy regions: $2 \leq E \leq 6$ MeV (below) and for $13 \leq E \leq 17$ MeV (above). The solid line is the GOE statistic curve and the dashed line is the Poisson one. Adapted from Ref. 9.

where $N(E)$ is the number of levels between E and zero for positive energy, between $-E$ and zero for negative energy. $\Delta_3(L)$ provides a measure of the degree of rigidity of the spectrum: for a given interval L, the smaller $\Delta_3(L)$ is, the stronger is the rigidity, signifying the long–range correlations between levels. For this statistics in the GOE ensemble

$$\Delta_3(L) = \begin{cases} \frac{L}{15}, & L \ll 1 \\ \\ \frac{1}{\pi^2} \ln L, & L \gg 1 \end{cases} . \tag{13}$$

In Figure 3 the spectral statistics $P(s)$ and Δ_3 are plotted for ^{166}Er. These statistics confirm the classical results: for energies above the saddle energy (~ 4 MeV) there is prevalently chaotic behaviour; for higher energies there is mixed behaviour.

4 Shell Model Calculations

In this section we discuss the statistical analysis of the shell–model energy levels in the $A = 46$–50 region. Exact calculations are performed in the ($f_{7/2}$,$p_{3/2}$,$f_{5/2}$,$p_{1/2}$) shell–model space, assuming ^{40}Ca as an inert core[11]. Diagonalizations are performed in the m–scheme using a fast implementation of the Lanczos algorithm with the code ANTOINE. For a fixed number of valence protons and neutrons we calculate the energy spectrum for projected total angular momentum J and total isospin T. The interaction we use is a minimally modified Kuo–Brown realistic force with monopole improvements.

We calculate the $T = T_z$ states from $J = 0$ to $J = 9$ for all the combinations of 6 active nucleons, i.e. ^{46}V, ^{46}Ti, ^{46}Sc, and ^{46}Ca, and also for ^{48}Ca and ^{50}Ca.

Since we are looking for deviations from chaotic features, we are mainly interested in the low–lying levels, up to a few MeV above the JT yrast line. Let us consider the energy levels up to 4, 5 and 6 MeV above the yrast line, and calculate the fluctuations around the average spacing between neighboring levels. In this range of energies, the level spectrum can be mapped into unfolded levels with quasi–uniform level density by using the constant temperature formula. To guarantee that the results up to different energies are unaffected by the unfolding procedure, the unfolding is performed using always the whole set of levels up to 6 MeV, for each JT set in the nucleus.

The mean level density can be assumed to be of the form

$$\bar{\rho}(E) = \frac{1}{T} \exp\left[(E - E_0)/T\right], \tag{14}$$

where T and E_0 are constants. For fitting purposes it is better to use not $\bar{\rho}(E)$ but its integral $\bar{N}(E)$. We write

$$\bar{N}(E) = \int_0^E \bar{\rho}(E')dE' + N_0 = \exp\left[(E - E_0)/T\right] - \exp\left[-E_0/T\right] + N_0. \tag{15}$$

The constant N_0 represents the number of levels with energies less than zero. We consider this function as an empirical function to fit the data and let N_0 take non–zero values. The parameters T, E_0 and N_0 that best fit $N(E)$ are obtained by minimizing the function

$$G(T, E_0, N_0) = \int_{E_{min}}^{E_{max}} [N(E) - \bar{N}(E)]^2 dE, \tag{16}$$

where $N(E)$ is the number of levels with energies less than or equal to E. The energies E_{min} and E_{max} are taken as the first and last energies of the level sequence.

Figure 4: $P(s)$ distribution for low–lying levels of fp shell nuclei with $0 \leq J \leq 9$: (a) ^{46}V, ^{46}Ti and ^{46}Sc; (b) ^{46}Ca, ^{48}Ca and ^{50}Ca. The dotted, dashed and solid curves stand for GOE, Poisson and Brody distributions, respectively. Adapted from Ref. 11.

As previously discussed, the spectral statistic $P(s)$ may be used to study the local fluctuations of the energy levels. $P(s)$ is the distribution of nearest–neighbour spacings $s_i = \tilde{E}_{i+1} - \tilde{E}_i$ of the unfolded levels.

For quantum systems whose classical analogs are integrable, $P(s)$ is expected to follow the Poisson limit, i.e. $P(s) = \exp(-s)$. On the other hand, quantal analogs of chaotic systems exhibit the spectral properties of GOE with $P(s) = (\pi/2)s \exp(-\frac{\pi}{4}s^2)$.

To quantify the chaoticity of $P(s)$ in terms of a parameter, it can be compared to the Brody distribution,

$$P(s,\omega) = \alpha(\omega+1)s^{\omega} \exp(-\alpha s^{\omega+1}), \tag{17}$$

with

$$\alpha = (\Gamma[\frac{\omega+2}{\omega+1}])^{\omega+1}. \tag{18}$$

This distribution interpolates between the Poisson distribution ($\omega = 0$) of

integrable systems and the GOE distribution ($\omega = 1$). The parameter ω can be used as a simple quantitative measure of the degree of chaoticity.

The number of $J = 0$–9 spacings below 4, 5 and 6 MeV range from 42, 66 and 105 in ^{46}Ca, to 86, 149 and 231 in ^{46}Ti, respectively.

To obtain a better estimate of the Brody parameter, we can combine spacings of different nuclei. The number of level spacings is now sufficiently large to yield meaningful statistics and we see that Ca isotopes are not very chaotic at low energy, in contrast to other nuclei in the same region (Figure 4).

Why are Ca isotopes less chaotic than their neighbors? We observe that the two–body matrix elements of the proton–neutron interaction are, on average, larger than those of the proton–proton and neutron–neutron interactions. Consequently the single–particle mean–field motion in nuclei with both protons and neutrons in the valence orbits suffers more disturbance and is thus more chaotic.

5 Conclusions

We have shown that in the roto–vibrational model of atomic nuclei, an order–chaos–order transition occurs as a function of the energy. Concerning the shell model calculations, the main conclusion of this paper is that for Ca isotopes we find significant deviations from the predictions of the random–matrix theory.

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